

Analytical design of...

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and 
$$\sum_{i=1}^n \frac{\partial v(x, \eta)}{\partial x_i} \frac{\partial \varphi_i(x, \xi, \eta)}{\partial \xi} + \frac{\lambda}{2} \sum_{i,j=1}^n \frac{\partial^2 v(x, \eta)}{\partial x_i \partial x_j} k_{ij} \sigma_i \sigma_j \frac{\partial (\mu_i \mu_j)}{\partial \xi} + \frac{\partial \omega}{\partial \xi} = 0 \quad (1.9)$$

The optimum functions  $v^0$  and  $\xi^0$  are determined by equations (1.8) and (1.9), whereby the conditions of the problem are satisfied by that solution of the equations, for which  $v^0$  is a positive definite form. In general, the solution of Eqs. (1.8) and (1.9) is very cumbersome. Hence, the following approximate method is proposed: Instead of system (1.2)(1.3), the auxiliary system

$$\frac{dx_i}{dt} = \psi_i[x, \eta, \theta], \quad \xi = \xi(x, \eta, \theta) \quad (2.1)$$

is considered, where the parameter  $\theta$  is introduced, so that for  $\theta = 0$ , the functional

$$\int_0^\infty e(x, \xi, \theta) dt$$

can be readily minimized, and that for  $\theta$  varying from zero to unity, the functions  $\psi$  and  $\xi$  pass continuously into the functions  $\varphi_i$  and  $\omega$ .

By differentiating Eqs. (1.8) and (1.9) with respect to  $\theta$ , equations can

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be obtained which describe the variation in the solutions  $\vartheta^0$  and  $\xi^0$  with varying  $\vartheta$ . In particular,  $\vartheta^0$  and  $\xi^0$  can be sought in the form of expansions:

$$\vartheta^0 = \sum_k a_k(\theta) \xi_k(x, \eta), \quad \xi^0 = \sum_k b_k(\theta) \xi_k(x, \eta) \quad (2.2) \quad \text{LH}$$

in terms of a system of functions  $\xi_k(x, \eta)$  which satisfies Eqs. (1.8) and (1.9) in approximating finite combinations of Eq. (2.2), (in the mean-square approximation, for example). The conditions of such an approximation yield the equations for the change in the coefficients  $a_k$  and  $b_k$  with respect to  $\vartheta$ . This method is also convenient by the fact that, proceeding from the stable solution of the problem for  $\vartheta = 0$ , and varying continuously the parameters of the problem and the solution with  $\vartheta$ , it is possible to obtain that branch of the solution to Eqs. (1.8) and (1.9) which also gives (for each  $\vartheta$ ) the solutions ensuring the passage of the trajectory of the transient process along the pre-assigned motion  $z_0(t)$  ( $x(t) = 0$ ). An example is given involving the choice of a con-

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troller for a second-order system. There are 4 Soviet-bloc references.

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16.4000 (1031)

AUTHORS: Krasovskiy, N.N., and Lidskiy, E. A. (Sverdlovsk)

TITLE: Analytical design of controllers in a system with chance properties. III. Optimum control in linear systems. Minimum mean-square error

PERIODICAL: Avtomatika i telemekhanika, v. 22, no. 11, 1961, 1425-1431

TEXT: The authors aim at determining an optimum control law for linear stochastic control systems and analyze conditions, in which a solution is possible. It is assumed that the automatic control system is defined by a linear motion equation in terms of usual coordinates, a variable  $\eta(t)$  describing chance properties of the system which is of a Markov character, and of a random pulse function representing interference. Transients are evaluated by a minimizing integral, and the problem is, therefore, that of finding a control law which would ensure a probable asymptotic motion stability of  $X = 0$ . The method used is based on Lyapunov's optimal

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functions  $v^0$ ; one of the conditions which  $v^0$  must satisfy is the Silvester criterion. Two cases are considered: A) When the variable  $\eta(t)$  can assume a finite number of values, B) When the function  $q(\eta, \beta)$  [Abstractor's note: Not defined in the present paper\_7 has

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a density  $p(\eta, \beta)$ . The function  $v^0$  is of the form  $v^0 = \sum_{i,j=1}^n b_{ij}(\eta) x_i x_j$ .

The coefficients  $b_{ij}(\eta)$  are determined by a system of algebraic equations in the case A), and in the case B) a system of integral equations can be obtained which extends to stochastic systems the equations first derived by A. M. Lyetov (Ref. 4: Avtomatika i telemechanika, v. 21, no. 4, 5, 6, 1960 and v. 22, no. 4, 1961). The method for approximate solution was previously described by the authors. It is illustrated in the case of the system defined by a linear motion equation. A parameter  $\vartheta$  is introduced;  $\vartheta = 0$  when there are  $n$  independently controlled channels, and  $\vartheta = 1$  when only one control affects all coordinates. When  $\vartheta = 0$ , the optimal func-

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tion is of the form  $v^0 = \sum_{i=1}^n b_{ii} x_i^2$ . When  $v^0 > 0$ , a system of differential equations is obtained of the form:

$$\frac{\partial b_{ij}^{(1)}}{\partial \vartheta} = \varphi_{ij}^{(1)}(b_{11}^{(m)}, \dots, b_{nn}^{(m)}, \vartheta) \quad (i=1, \dots, n; j=1; m=1, \dots, k) \quad (4.6)$$

These equations must be integrated in the interval  $0 \leq \vartheta \leq 1$  taking into account the initial conditions, at  $\vartheta = 0$ . The question whether the integration is possible is equivalent to the problem of existence of an optimum solution. Assuming the existence of a permissible control signal it is proved that an optimum control also exists. The existence of a permissible signal ensures the convergence of the integral of the mean-square error if the system is linear. The proof of the possibility of finding optimum control is based on

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auxiliary systems with the parameter  $\theta$ ; it is assumed that interference depends only on mismatch. An equation is derived in which the left-hand is a function of the mathematical expectation of optimal function and the right-hand side is a quadratic expression. There are 5 Soviet-bloc references. 4

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16.8000 (103,1344)

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AUTHORS: Krasovskiy, N.N., and Lidskiy, E.A. (Sverdlovsk)

TITLE: Analytic construction of regulators in stochastic systems with integrals on the velocity of change of the regulating influence

PERIODICAL: Akademiya nauk SSR. Otdeleniye tekhnicheskikh nauk. Prikladnaya matematika i mekhanika, v. 25, no. 3, 1961, 420 - 432

TEXT: A regular system is considered, in which the transition process may be written in stochastic differential equations of perturbed motion

$$\frac{dx_i}{dt} = f_i [x_1, \dots, x_n, \xi, \eta(t)] \quad (i = 1, \dots, n) \quad (1.1)$$

where  $x_i$  are the deviations of the actual values of the coordina-

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tes of the regulated vector quantity from the original (unperturbed) value  $x_1 = 0$  ( $i = 1, \dots, n$ )  $\xi$  is the regulating influence,

Since the system is subject to random change, the random variable  $\eta(t)$  is included in the arguments of the function. It is assumed that  $f_i$  are known continuous functions satisfying the Lipschitz conditions [Abstractor's note: Conditions not stated] in some region  $G$  of the space  $(x, \xi, \eta)$ ,  $f_1(0, \dots, 0, \eta(t)) = 0$ , and the variable  $\eta(t)$  describes the Markov random process [Abstractor's note: Process not stated]. The correct formulation of  $\xi$  in the regulator is obtained from

$$J = \int_0^{\infty} M \{ \omega [x_1(t), \dots, x_n(t), \xi(t), \dot{\xi}(t)] \} dt = \min \quad (1.2)$$

where  $M$  denotes the expectation of the random quantity, which is a given non-negative function of the arguments. The equation of the optimum regulator is of the form

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$$\dot{\xi} = [x_1, \dots, x_n, \xi, \eta]. \quad (1.3)$$

The realization of the random variable  $\eta(t)$  is a power function  $\eta^{(p)}(t)$ . If  $\eta$  in (1.3) is chosen then for each realization and initial condition  $(x_{10}, \xi_0, t = t_0)$ , (1.1) and (1.3) have a continuous realization  $x^{(p)}(x_0, \xi_0, t_0, t, \eta^{(p)})$ ,  $\xi^{(p)}(x_0, \xi_0, t_0, t, \eta^{(p)})$  of  $x(t)$ ,  $\xi(t)$ . If the random process takes place in the space  $(x_1, \dots, x_n, \xi, \eta)$ , then it is described as the Markov random process. Let  $\{x(t), \xi(t), \eta(t)/x_0, \xi_0, \eta_0, t_0\}$  denote a Markov random vector-function with initial conditions  $x_1, x_{10}, \xi = \xi_0, \eta = \eta_0$  at  $t = t_0$ , and giving for  $t \geq t_0$  a solution of (1.1) and (1.3). It is assumed that  $f_1$  and  $\xi$  are continuous in the space  $(x, \xi)$  and that the region  $G$  is given by  $-\infty < x_i < \infty$  ( $i = 1, \dots, n$ ),  $-\infty$

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$\xi \in \mathbb{R}, \eta_1 \leq \eta \leq \eta_2$ .  $J_\xi$  is given by

$$J_\xi[x_0, \xi_0, \eta_0] = \int_0^\infty M(\omega[x(t), \xi(t), \dot{\xi}(t)]/x_0, \xi_0, \eta_0, t_0 = 0) dt \quad (2.3)$$

The problem consists of determining  $\xi^0[x, \xi, \eta]$ , satisfying a)  
(2.1.) The unperturbed motion  $x = 0, \xi = 0$  with  $\dot{\xi} = \xi^0$  must be asymptotic to the probability relative to any initial perturbation.  
b) Condition (2.2.). For any initial conditions (2.3) must be the final condition and

$$J_{\xi^0}[x_0, \xi_0, \eta_0] = \min_{\xi} J_\xi[x_0, \xi_0, \eta_0] \quad (2.4)$$

must hold. The problem is solved by means of the optimum Lyapunov function  $v^0$  which is required to satisfy the following conditions:  
Condition 3.1.

$$v^0(x, \xi, \eta) \geq w(x, \xi) > 0 \text{ for } (x, \xi) \neq 0$$

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$$w(x, \xi) \rightarrow \infty \text{ for } \{x, \xi\} \rightarrow \infty;$$

$$\text{Condition 3.2. } \left( \frac{dM \{v^0\}}{dt} \right) = - \omega[x_1, \dots, x_n, \xi, \xi^0]; \quad (3.1)$$

$$\text{Condition 3.3. } \left( \frac{dM \{v^0\}}{dt} \right)_{\xi^0} + \omega[x_1, \dots, x_n, \xi, \xi^0] = \min_{\xi} \left[ \left( \frac{dM \{v^0\}}{dt} \right)_{\xi} + \omega[x_1, \dots, x_n, \xi, \xi] \right]; \quad (3.2)$$

Condition 3.4.:  $v^0(x, \xi, \eta)$  may have an infinitely small upper bound, and  $v^0[x, \xi, \eta] \rightarrow 0$  as  $\omega[x, \xi, \eta] \rightarrow 0$ . The solutions for  $v^0$  and  $\xi^0$  are given by

$$\sum_{i=1}^n \frac{\partial v^0(x, \xi, \eta)}{\partial x_i} f_i(x, \xi, \eta) + \frac{\partial v^0(x, \xi, \eta)}{\partial \xi} \zeta^0(x, \xi, \eta) + \sum_{k=1}^m v^0(x, \xi, \beta_k) q(\eta, \beta_k) - q(\eta) v^0(x, \xi, \eta) + \omega[x, \xi, \zeta^0] = 0 \quad (4.7)$$

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$$\frac{\partial v^0(x, \xi, \eta)}{\partial \xi} + \frac{\partial \omega(x, \xi, \zeta^0)}{\partial \zeta} = 0 \quad (4.8)$$

where  $q = 1-p$  has its usual significance in the probability theory. These equations may be solved by approximation methods. The problem of choice of parameters of the linear system minimizing the quadratic criterion of quality is then discussed. From the works of N.G. Chetayev (Ref. 2: Ustoychivost' dvizheniya (Stability of Motion), Gostekhizdat M., 1956), A.A. Krasovskiy and A.A. Fel'dbaum (Ref. 14: Vychislitel'nyye ustroystva v avtomaticheskikh sistemakh (Calculating Structure in Automatic Systems) Fizmatgiz, 1959) and J. Bertram and R. Kalman the following equations are obtained

$$\sum_{i=1}^2 \frac{\partial v^0(x, \xi, \eta_i)}{\partial x_i} \left[ \sum_{j=1}^2 a_{ij}(\eta_i) x_j + m_i \xi \right] + p_{ik} [v^0(x, \xi, \eta_k) - v^0(x, \xi, \eta_i)] - \frac{1}{4} \left[ \frac{\partial v^0(x, \xi, \eta_i)}{\partial \xi} \right]^2 = -x_1^2 - x_2^2 - \xi^2 \quad (l=1,2; k \neq l) \quad (6.6)$$

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$$v^l(x_1, x_2, \xi, \eta_l) = \sum_{i,j=1}^2 [b_{ij}(\eta_l) x_i x_j + b_i(\eta_l) x_i \xi] + c(\eta_l) \xi^2 \quad (l=1, 2) \quad (6.7)$$

By substituting the right-hand side of (6.7) in (6.6) and equating coefficients of like powers of  $x_1$  and  $\xi$ , the equations for  $b_{ij}$ ,  $b_i$  and  $c$  may be obtained as required. The problem of the existence of an optimum control law for the case

$$P[x_i \rightarrow x_j \text{ after time } \Delta t] = p_{ij} \Delta t + o(\Delta t) \quad (i \neq j). \quad (6.3)$$

This is the question of the existence of a solution  $v^0$  of the system (6.6). A regularization law  $\xi = \xi(q)$  is said to be possible if by this law the system

$$\frac{dx_1}{dt} = \sum_{j=1}^2 a_{1j}(\eta) x_j + m_1 \xi \quad (j = 1, 2) \quad (6.1)$$

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approaches the probability asymptotically, and the integral of (6.3) has a finite value for any initial conditions. The problem of the existence (and optimum properties) of a control is investigated by F.M. Kirillova. In the case where a possible control exists, the existence of an optimum control is investigated by considering the system

$$\frac{dx_i}{dt} = \vartheta \left[ \sum_{j=1}^2 a_{ij}(\eta) x_j + m_i \xi \right] - (1 - \vartheta)x_i \quad (i = 1, 2; 0 \leq \vartheta \leq 1), \quad (7.1)$$

and

$$\dot{\xi} = \zeta[x_1, x_2, \xi, \eta; \vartheta]. \quad (7.2)$$

It is required that for every  $\vartheta \in [0, 1]$  the system is asymptotic to the probability and that

$$J_{\zeta}[x_{10}, x_{20}, \xi_0, \eta_0, \vartheta] =$$

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$$= \int_0^{\infty} M \{ [x_1^2(t) + x_2^2(t) + \xi^2(t) + \xi^2(t)] / x_{10}, x_{20}, \xi_0, \eta_0, \tau_0 = 0 \} dt = \min \quad (7.3)$$

The optimum condition is found to be  $\gamma = 1$ . There are 15 references: 13 Soviet-bloc and 2 non-Soviet-bloc. The references to the English-language publications read as follows: R. Bellman, Dynamic programming and stochastic control processes. Inform. and Control, vol. 5, p 228-239, 1958; R.E. Kalman and I.E. Bertram, Control System Analysis and Design Via the "Second Method" of Lyapunov. Paper Amer. Soc. Mech. Engrs., no. 2, 1959.

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S/040/61/025/004/009/021  
D274/D306

16.3400

AUTHORS:

Klimushev, A.I. and Krasovskiy, N.N. (Sverdlovsk)

TITLE:

Uniform asymptotic stability of differential-equation systems with a small parameter in front of the derivatives

PERIODICAL:

Prikladnaya matematika i mekhanika, v. 25, no. 4, 1961, 680-690

TEXT: The asymptotic stability of a system of differential equations which have a small parameter in front of the derivatives, is proved on the basis of uniform asymptotic stability of a degenerate system of first-approximation differential and of asymptotic stability of some auxiliary system. The method of Lyapunov-functions is used. A linear system is considered

$$\frac{dx_i}{dt} = \sum_{l=1}^m a_{il}(t)x_l + \sum_{s=1}^n b_{is}(t)y_s + f_i(t) \quad (i = 1, \dots, m)$$

(1.1)

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$$\mu \frac{dy_j}{dt} = \sum_{l=1}^m c_{jl}(t)x_l + \sum_{s=1}^n d_{js}(t)y_s \quad (j = 1, \dots, n)$$

where  $\mu$  is a small positive parameter. The stability of the solutions

$$x_i = x_i(t, \mu), \quad y_j = y_j(t, \mu) \quad (1.3)$$

is investigated. It is assumed that the coefficients in Eq. (1.1) are bounded continuous functions of  $t$ , having bounded continuous derivatives. An additional condition is imposed on  $d_{js}(t)$ . For  $\mu = 0$ , system (1.1) degenerates into

$$\frac{dx_i}{dt} = \sum_{l=1}^m a_{il}(t)x_l + \sum_{s=1}^n b_{is}(t)y_s + f_i(t), \quad \sum_{l=1}^m c_{jl}(t)x_l + \sum_{s=1}^n d_{js}(t)y_s = 0 \quad (1.5)$$

Solving the second part of (1.5) with respect to  $y$ , and substituting the solutions in the first  $m$  equations of (1.5), one obtains

$$\frac{dx_i}{dt} = \sum_{k=1}^m r_{ik}(t)x_k + f_i(t) \quad (i = 1, \dots, m) \quad (1.8)$$

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where  $r_{ik}(t)$  are bounded continuous functions. The system

$$\frac{dy_j}{dt} = \sum_{s=1}^n d_{js}(t)y_s \quad (j = 1, \dots, n) \quad (1.9)$$

is called auxiliary. It is shown that for sufficiently small positive values of the parameter  $\mu$  (and under certain conditions), the trajectory of system (1.1) tends to the trajectory of (1.5), and that from the stability of the auxiliary system follows the stability of (1.1). Theorem 1.1. If for (1.1) and (1.5) the following conditions hold: 1) system (1.8) is uniformly asymptotically stable,

2) the system  $\frac{dy_j}{dt} = \sum_{s=1}^n d_{js}(\theta)y_s \quad (j = 1, \dots, n)$

with constant coefficients is (for all fixed values of  $\theta$ ) uniformly asymptotically stable (with respect to  $\theta \in [t_0, \infty]$ ), then for sufficiently small  $\mu$  the system (1.1) is uniformly asymptotically stable, and for any pre-assigned positive  $Q$  and  $\epsilon$  it is possible to find a positive  $\mu_0$ , so that

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$$|x_i(t, \mu) - x_i(t)| < \varepsilon, \quad |y_j(t, \mu) - y_j(t)| < \varepsilon \quad (1.10)$$

$$|y(t_0, \mu) - y_j(t_0)| < Q \text{ where } t > t_1(Q, \varepsilon)$$

holds if  $\mu < \mu_0$ .  $\mu_0$  is chosen as small as required for making the difference between  $t_1$  and  $t_0$  arbitrarily small. The proof of the theorem is given. The asymptotic stability of (1.1) is verified by considering two solutions with different initial conditions; in the verification, Lyapunov's function is used. On non-linear systems, system

$$\frac{dx_i}{dt} = X_i(x_s, y_k, t), \quad x_{i0} = a_{i0} \quad (i, s = 1, \dots, m)$$

$$\mu \frac{dy_j}{dt} = Y_j(x_s, y_k, t), \quad y_{j0} = b_{j0} \quad (j, k = 1, \dots, n) \quad (2.1)$$

is considered, as well as

$$\frac{dx_i}{dt} = X_i[x_s, f_k(x_s, t), t] = F_i(x_s, t), \quad x_{i0} = a_{i0} \quad (2.4)$$

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obtained from the degenerate system (analogous to that of the first part of the article). An auxiliary system (of equations of perturbed motion) is set up. It is assumed that the linear approximation to the perturbed system

$$\frac{dz_i}{dt} = \sum_{s=1}^n g_{is}(t) z_s \quad \left( g_{is}(t) = \left[ \frac{\partial X_i}{\partial x_s} + \sum_{k=1}^n \frac{\partial X_i}{\partial y_k} \cdot \frac{\partial f_k}{\partial x_s} \right]_{(x_s=x_s(t))} \right) \quad (2.8)$$

is uniformly asymptotically stable. In addition, the system

$$\frac{dy_j}{dt} = Y_j(\alpha_s, y_k, \beta) \quad (2.9)$$

is considered. The following theorem is proved: Theorem 2.1. If for the system (2.1) the following conditions hold: 1) system (2.8) is uniformly asymptotically stable, 2) for the system (2.9) it is possible to find (for any fixed  $\alpha_s$  and  $\beta$ ) constant symmetrical matrices so that the matrix obtained by symmetrization of these matrices has negative eigenvalues  $r_1 < -\gamma$  ( $\gamma$  is a positive con-

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stant), then (for sufficiently small  $\mu$ ) the solution of system (2.1) is uniformly asymptotically stable with respect to small deviations of  $x_{i0}$  and deviations of  $y_{j0}$ . There are 6 Soviet-bloc references.

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KRASOVSKIY, N.N. (Sverdlovsk)

Optimum RMS stabilization in the case of random damped  
excitations. Prikl. mat. i mekh. 25 no.5:806-817 S-O '61.  
(Automatic control) (MIRA 14:10)

89527

S/039/61/053/002/002/003  
C111/C222

/6.9500 (1031, 1121, 1122)

AUTHOR: Krasovskiy, N.N. (Sverdlovsk)

TITLE: On a method for constructing optimal trajectories <sup>η</sup>

PERIODICAL: Matematicheskii sbornik, vol.53, no.2, 1961, 195-206

TEXT: The author considers the system

$$\frac{dx}{dt} = P(t)x + b\eta + \varphi(t), \quad (0.1)$$

where  $x = \{x_i\}$  -- n-dimensional position vector of the image point  $x$  in the phase space,  $b = \{b_1, \dots, b_n\}$  -- constant vector,  $P(t)$  -- an  $n \times n$  matrix with continuous elements  $p_{ij}(t)$ ,  $t \in (0, \infty)$ ;  $\varphi(t) = \{\varphi_1(t), \dots, \varphi_n(t)\}$  the continuous outer action,  $\eta(t)$  -- scalar control. ✓

Problem A: For a given  $x = x_0$ ,  $t = t_0 = 0$  determine a piecewise smooth  $\eta^0(t)$  which is denoted as an optimal control so that it satisfies the condition

$$|\eta(t)| \leq 1 \quad (0.2)$$

and that the trajectory  $x(x_0, t, \eta^0)$  of (0.1) for  $\eta = \eta^0(t)$  reaches the Card 1/10

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point  $x = 0$  in the shortest time  $t = T^0$ .

Problem B: For a given  $x = x_0$ ,  $t = t_0 = 0$ ,  $\eta^0(t)$  shall be determined

so that (0.2) is satisfied and the trajectory  $x(x_0, t, \eta^0, M)$  of (0.1)

reaches the given manifold  $M$  of the space  $x$  in the shortest time.

The author proposes an effective way of solution for which the problems  $A$  and  $B$  are approximated by problems the solutions of which are smoother, and which depend on certain parameters.

Beside of (0.1) the author considers the auxiliary system

$$\frac{dx}{dt} = A(t)x + B(\mu)u + \varphi(t), \quad (1.1)$$

where  $0 \leq \mu \leq 1$ ,  $0 \leq \mu \leq 1$ ,  $B(\mu)$  is an  $n \times n$  matrix with the elements

$$b_{i1} = b_i \quad (i=1, \dots, n), \quad (1.2)$$

$$b_{ij} = \xi_{ij} \quad (i=1, \dots, n; j=2, \dots, n), \quad (1.3)$$

where  $\xi_{ij}$  are chosen so that  $B(\mu)$  does not degenerate for  $\mu \in (0, 1]$ .

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Problem  $A_{\mathcal{S}, \mu}$ : Determine the optimal control  $u^0(t, \mathcal{S}, \mu)$  for  $x = x_0$ ,  
 $t = t_0 = 0$  so that it holds

$$u_1^2(t) + \dots + u_n^2(t) \leq 1, \quad (1.4)$$

and the trajectory  $x(x_0, t, u^0)$  of (1.1) for  $u^0(t, \mathcal{S}, \mu)$  reaches the  
point  $x = 0$  in the shortest time  $t = T^0(\mathcal{S}, \mu)$ .

The problem  $B_{\mathcal{S}, \mu}$  is defined analogously.

Let  $F(t, \mathcal{S})$  be the fundamental matrix of the solutions of

$$\begin{aligned} \frac{dx}{dt} &= \mathcal{S}P(t)x \\ (F(0, \mathcal{S}) &= E); \end{aligned} \quad (1.5)$$

and let

$$H(t, \mathcal{S}, \mu) = F^{-1}(t, \mathcal{S})B(\mu). \quad (1.6)$$

If the problem  $A_{\mathcal{S}, \mu}$  is reduced to the "L-problem" (Ref.11: N.N.Krasovskiy,  
Ob odnoy zadache optimal'nogo regulirovaniya [On a problem of optimal

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On a method for constructing...

controls] Priklad. matem. i mekh., vol. 21, no. 5 (1957), 670-677) then  
it follows: The problem  $A_{\mathcal{B}, \mu}$  has a unique solution;  $T^0(\mathcal{B}, \mu)$  is the  
least root of

$$\lambda(T, \mathcal{B}, \mu) = \min \int_0^T \left[ \sum_{j=1}^n \left[ \sum_{i=1}^n \lambda_i h_{ij}(\tau, \mathcal{B}, \mu) \right]^2 \right]^{\frac{1}{2}} d\tau = 1 \quad (2.4)$$

$$\left( \sum_{i=1}^n c_i \lambda_i = 1 \right),$$

where  $c_i$  are components of

$$c = -x_0 - \int_0^T \mathcal{B} F^{-1}(\tau, \mathcal{B}) \varphi(\tau) d\tau \quad (2.2)$$

$$(\mathcal{B} = 1),$$

with  $0 \leq \mathcal{B} \leq 1$ ; the optimal control is given by

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On a method for constructing...

$$u_j^0(t, \vartheta, \mu) = \frac{\sum_{i=1}^n \lambda_i^0 h_{ij}(t, \vartheta, \mu)}{\left[ \sum_{j=1}^n \left[ \sum_{i=1}^n \lambda_i^0 h_{ij}(t, \vartheta, \mu) \right]^2 \right]^{\frac{1}{2}}} \quad (2.5)$$

$(j = 1, \dots, n).$

where  $\lambda_i^0(\vartheta, \mu)$  is a solution of (2.4) and the denominator of (2.5) is  $\neq 0$  since  $B(\mu)$  does not degenerate.  
It is stated:  
Lemma 2.1. It holds

$$\lim_{\mu \rightarrow 0} T^0(1, \mu) = T^0. \quad (2.6)$$

Lemma 2.2. For  $\mu \rightarrow 0$  the first component  $u_1^0(t, 1, \mu)$  of the optimal control of the problem  $A_{1\mu}$  tends with respect to the measure to the optimal control  $\eta^0(t)$  of the problem A.

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Furthermore it is stated that locally optimal trajectories are possible for  $\varphi(t) \neq 0$ :

Definition 3.1. The trajectory  $x(x_0, t, u^0)$  of (1.1) is locally optimal if there exists an  $\varepsilon > 0$  so that there exists no admissible control  $u(t)$  which satisfies (1.4), and for which  $x(x_0, t, u)$  reaches the point  $x = 0$  for  $t = T < T^0(\varphi, \mu)$ , where

$$\sum_{i=1}^n [x_i(x_0, t, u) - x_i(x_0, t, u^0)]^2 \leq \varepsilon^2 \quad (0 \leq t \leq T) \quad (3.2)$$

is satisfied.

From the lemmas 2.1 and 2.2 and some further ones on the locally optimal trajectories there results the following way of solution: (1.1) is constructed; for  $\lambda = 0, \mu = 1$  the optimal trajectory between  $x = x_0$

and  $x = 0$  is a straight line. Having the solution for a  $\lambda \geq 0 (\mu = 1)$  then one uses the theorems on implicit functions and determines

$\frac{dT^0}{d\lambda}, \frac{d\lambda_1^0}{d\lambda}$ . The integration of the obtained differential equations

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permits the continuation of the optimal solution to greater  $\lambda$ -values.  
After reaching  $\lambda^* = 1$ , analogous equations are established for  $\frac{dT^0}{d\lambda}$  and

$\frac{d\lambda_i^0}{d\lambda}$  and integrated for  $\lambda \in (0, 1]$ .

Let

$$\gamma(T, \lambda; \mu, \{\lambda_j\}) = \int_0^T \left[ \sum_{j=1}^n \left( \sum_{i=1}^n \lambda_i h_{ij} + \frac{1}{c_1} \left( 1 - \sum_{k=2}^n c_k \lambda_k \right) h_{1j} \right)^2 \right]^{\frac{1}{2}} d\tau. \quad (4.4)$$

Theorem 4.1. Let  $T^0(\lambda, \mu)$ ,  $\lambda_i^0(\lambda, \mu)$  ( $0 \leq \lambda \leq 1$ ;  $0 < \mu \leq 1$ ,  $\lambda^* = 1$ ) be solutions of the system

$$\frac{dT^0}{d\lambda} = - \frac{\frac{\partial \lambda}{\partial T}}{\frac{\partial \lambda}{\partial T}}, \quad (4.2)$$

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$$\frac{d\lambda_i^0}{d\theta} = - \frac{D \left[ \frac{\partial \gamma}{\partial \lambda_1}, \dots, \frac{\partial \gamma}{\partial \lambda_n} \right]}{D [\lambda_1, \dots, \lambda_n]} \quad (4.7)$$

$$(i = 2, \dots, n)$$

$$\frac{dT^0}{d\mu} = - \frac{\frac{\partial \gamma}{\partial \mu}}{\frac{\partial \gamma}{\partial T}} \quad (\theta = 1), \quad (4.8)$$

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$$\frac{d\lambda_i^0}{d\mu} = - \frac{\frac{D \left[ \frac{\partial \gamma}{\partial \lambda_2}, \dots, \frac{\partial \gamma}{\partial \lambda_n} \right]}{D [\lambda_2, \dots, \mu_i, \dots, \lambda_n]}}{\frac{D \left[ \frac{\partial \gamma}{\partial \lambda_2}, \dots, \frac{\partial \gamma}{\partial \lambda_n} \right]}{D [\lambda_2, \dots, \lambda_n]}} \quad (4.9)$$

( $i = 2, \dots, n; \mu = 1$ ).

where

$$\frac{\partial \lambda}{\partial T} > \alpha > 0 \quad (\alpha = \text{const}) \quad (4.10)$$

be satisfied on these solutions. Then

$$T^0 = \lim_{\mu \rightarrow 0} T^0(i, \mu) \quad (4.11)$$

is the optimal controlling time of the problem A; to the time  $T^0$  there corresponds the local-optimal trajectory with the optimal control

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$$\eta^0(t) = \text{sign} \left[ \sum_{i=1}^n \lambda_i h_{i1}(t) \right], \quad \lambda_i = \lim_{\mu \rightarrow 0} \lambda_i^0(1, \mu). \quad (4.12)$$

Then the problem B is treated, where the equations (4.2), (4.7)-(4.9) are completed by the equations which describe the variation of the coordinates  $\zeta_i^0(\lambda, \mu)$  of the point  $\zeta^0 \in M$  in dependence of  $\lambda, \mu$ . ✓

The author mentions A.A.Fel'dbaum, L.S.Pontryagin, V.G.Boltyanskiy, R.V.Gamkrelidze, A.F.Filippov and F.M.Kirillova. There are 11 Soviet-bloc and 2 non-Soviet-bloc references. The two references to English-language publications read as follows: R.Bellman, J.Glicksberg, O.Gross, Some aspects of the mathematical theory of control processes, California, 1958. J.P.La Salle, Time optimal control systems, Proc.Nat.Acad. of Sc., 45, no.4 (1959), 573-577.

SUBMITTED: April 21, 1959

Card 10/10

KRASOVSKIY, N. N.

PHASE I BOOK EXPLOITATION

SOV/6201

Vsesoyuznyy s"yezd po teoreticheskoy i prikladnoy mekhanike. 1st, Moscow, 1960.

Trudy Vsesoyuznogo s"yezda po teoreticheskoy i prikladnoy mekhanike,  
27 yanvarya -- 3 fevrallya 1960 g. Obzornyye doklady (Transactions of the  
All-Union Congress on Theoretical and Applied Mechanics, 27 January to  
3 February 1960. Summary Reports). Moscow, Izd-vo AN SSSR, 1962.  
467 p. 3000 copies printed.

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Transactions of the All-Union Congress (Cont.)

SOV/6201

**PURPOSE:** This book is intended for scientific and engineering personnel who are interested in recent work in theoretical and applied mechanics.

**COVERAGE:** The articles included in these transactions are arranged by general subject matter under the following heads: general and applied mechanics (5 papers), fluid mechanics (10 papers), and the mechanics of rigid bodies (8 papers). Besides the organizational personnel of the congress, no personalities are mentioned. Six of the papers in the present collection have no references; the remaining 17 contain approximately 1400 references in Russian, Ukrainian, English, German, Czechoslovak, Rumanian, French, Italian, and Dutch.

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- Bogolyubov, N. N., and Yu. A. Mitropol'skiy. Analytic Methods of the Theory of Nonlinear Oscillations 25

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Krasovskiy, N. N. Lyapunov's Second Method in the Theory  
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Rumyantsev, V. V. Stability of Motion of a Solid Body With Fluid-Fixed  
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Babenko, K. I. Numerical Methods for Solving Problems of  
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16 8000 (also 3823, 4023)

38830  
S/103/62/023/006/001/012  
D230/D308

AUTHORS:

Krasovskiy, N.M. and Letov, A.M. (Sverdlovsk. Moscow)

TITLE:

Theory of the analytical design of regulators

PERIODICAL:

Avtomatika i telemekhanika, v. 23, no. 6, 1962,  
715-720

TEXT:

A detailed examination of the problem already con-  
sidered by A.M. Letov (Avtomatika i telemekhanika, v. 22, no. 4,  
1961), concerning the analytical design of an optimum regulator  
 $u^0[x]$ , which minimizes the functional

$$J[x, u] = \int_0^{\infty} \omega[x(x_0, t), u(t)] dt \quad (1)$$

for the motion  $x(x_0, t)$  of the system  $dx/dt = Ax + mu$ , where the sym-  
bols have their usual meaning. The scalar magnitude  $u$  describing  
the controlling action is conditioned by

$$|u| \leq K \quad (5)$$

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\* S/103/61/022/004/001/014

Theory of the analytical design ....

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The solution procedure is as follows: the optimum control  $u^*(x)$  is formulated for the functional and for the function of the system, without accounting for the limitation and then one assumes that

$$\begin{aligned} u^0(x) &= u^*(x) & \text{for } |u^*| &\leq N, \\ u^0(x) &= N, & \text{for } u^* &\geq N, \\ u^0(x) &= -N, & \text{for } u^* &\leq -N. \end{aligned} \quad (4)$$

The problem is analyzed in detail; conditions for the existence of the above assumptions are explained in terms of the dynamic programming theory. The determination of the optimum control  $u^0(x)$  using dynamic programming reduces to solving the Bellman equation for the continuous function  $v(x) = \min J[x, u]$ ; condition of continuity is established. The existence of the various complexities of the problem is established and analyzed further in detail by means of an example. There are 4 figures.

SUBMITTED: February 2, 1962

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16,8000 (1031, 1132)

32690  
S/04 O/62/026/001/005/023  
D237/D304

AUTHOR: Krasovskiy, N.N. (Sverdlovsk)

TITLE: On the analytical construction of optimal control in the system with a time lag

PERIODICAL: Akademiya nauk SSSR. Otdeleniye tekhnicheskikh nauk. Prikladnaya matematika i mekhanika, v. 26, no. 1, 1962, 39-51

TEXT: The author considers the problem of formulating the controlling action of the regulator in the system, in which the controlled object is described by linear differential equations with a time lag. Optimal control acts under the conditions of asymptotic stability of the given motion, and of the integral with respect to time of the square of deviation of coordinates of controlled magnitude and controlling action, being minimum. The solution is based on A.M. Laypunov's method (Ref. 1: Obshchaya zadacha ob ustoychivosti dvizheniya (General Problem of Stability of Motion) Gostekhizdat, 1950) extended to systems with a time lag and brought into agreement with the principles of dynamic programming. It is shown that

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On the analytical construction ...

optimal action of the regulator taking place at every moment  $t$  during the control, should be expressed as a linear functional of the functions, describing the behavior of the controlled magnitude during the preceding interval of time,  $t-h \leq \tau \leq t$  ( $h$  = time lag). A real form of this functional is given. The author then uses a method of deformation of the system to explain the problem of existence of the solution, and gives an approximate method of constructing optimal control. This paper is a generalization of the work of A.M. Letov (Ref. 6: Avtomatika i telemekhanika, 1961, v 22, no. 4). The author expresses his gratitude for the help obtained. There are 1 figure and 21 references: 19 Soviet-bloc and 2 non-Soviet-bloc. The references to the English-language publications read as follows: I. Kramer, Information and control, v. 3, no. 4 (1960); R. Bellman and R. Kalaba, Journal of Basic Engineering, March (1961).

SUBMITTED: August 14, 1961

Card 2/2

36031  
S/C40/62/026/002/003/025  
D299/D301

16.8000

AUTHOR: Krasovskiy, N.M. (Sverdlovsk)

TITLE: On a control problem

PERIODICAL: Prikladnaya matematika i mekhanika, v. 26, no. 2,  
1962, 218 - 232

TEXT: The problem is considered of selecting a controller function  $u$ , for which the motion  $z(t)$  takes place in the neighborhood of the point  $y(t)$ . The motion  $y(t)$  constitutes a random diffusion process, whereas  $z(t)$  is described by a linear differential equation which contains the controller function  $u$ . The value of  $u$  is determined at each moment  $\tau$  on the basis of the measured values  $y(\tau)$  and  $z(\tau)$ . It is assumed that it is possible to measure these values during the control process and to transmit instantaneously the results to the controller element. Such a formulation corresponds to optimum-control problems with measurable parameters and random disturbances. The sought-for controller  $u_0$  is restricted by the condition

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D299/0301

On a control problem

$$\int_0^T M\{(u^0[t, y(t), z(t)])^2; y(0), z(0); dt \cdot \infty. \quad (2.3)$$

The original systems of equations for  $y(t)$  and  $z(t)$  yield (after a change of variables  $x = z - y$ ), the equation

$$\dot{x}(t) = [A(t)(x(t) + y(t) - M(t, y(t)) + b(t)u]dt - M(t, y(t))dy(t) \quad (2.4)$$

The problem is reformulated as follows: The quantities  $T=0, \dots, 0$ ,  $x=0$  and  $p=1$  are given; it is required to find an allowed controller  $u^0(t, x, y)$ , which satisfies condition (2.3), so that if  $u = u^0$  in Eq. (2.4), then the condition

$$P[\|x(T)\| \leq \epsilon; x(0), y(0)] \leq p \quad (2.5)$$

$$\|x(0)\| \leq \epsilon, \|y(0)\| \leq \epsilon. \quad (2.6)$$

holds, if

Only the linear case is considered; the reasoning can be extended to the nonlinear case, yet the calculation of  $u^0$  becomes thereby difficult. The solution to the above problem is stated in the form of a theorem: Let, on the interval  $[0, T]$ , the controllability conditions (involving the linear independence of the vectors  $L_n$ ) be

On a control problem

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satisfied, the functions  $A(t)$ ,  $b(t)$ ,  $K(t)$  and  $R(t, y)$  be continuous, and the function  $R(t, y)$  also satisfy Lipschitz's condition and be bounded; then it is possible to construct, (for any positive  $\epsilon$  and  $N$ , and  $p < 1$ ), an allowed  $u^0$  which satisfies the conditions of the problem. This controller can be selected in the form of a linear function

$$u^0[t, x, y] = \sum_{i=1}^n [u_i(t)x_i + v_i(t)y_i], \quad (3.6)$$

where  $u$  and  $v$  are continuous functions. The determination of  $u$  and  $v$  reduces to the solution of Cauchy's problem for an ordinary system of differential equations. The problem under investigation is solved by a modernized method of Lyapunov functions. Thereby condition (2.5) is replaced by the stronger condition

$$M[//x(T)//^2; x(0), y(0)] \leq (1 - p)\epsilon^2 \quad (//x(0)//^2 + //y(0)//^2) \quad (4.2)$$

The method is based on the theory of dynamic programming. It is noted that the controller  $u^0$  in the stochastic problem, is constructed. ✓

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On a control problem

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D299/D301

ted analogously to that in a determinate problem. An example, illustrating the above theoretical considerations, is the problem of matching (during an interval of time  $T$ ), the controlled motion of a rotating body with the rotation of a body subjected to random moment of momentum. There are 20 references: 12 Soviet-bloc and 8 non-Soviet-bloc, (including 5 translations). The references to the English language publications read as follows: R. Bellman, Dynamic programming and stochastic control processes. Information and control, 1958, v. 1, 228-259; I.P. La Salle, Time optimal control systems. Proc. of the National Acad. of Sci. v. 45, 1958, no. 4; R.E. Kalman, New Methods and Results in Linear Prediction and Filtering Theory. RIAS Technical Report, 61-1.

SUBMITTED: December 21, 1961

Card 4/4

KRASOVSKI, N. N.

"Optimal Processes in Systems with Time Delay."

Paper to be presented at the IFAC Congress held in  
Basel, Switzerland, 27 Aug to 4 Sep 63

KRASOVSKIY, N. N.  
AID Nr. 984-4 / 6 June

OPTIMAL CONTROL IN LINEAR SYSTEMS WITH TIME DELAY (USSR)

Krasovskiy, N. N. Sibirskiy matematicheskiy zhurnal, v. 4, no. 2, Mar-Apr 1963, 295-302.  
S/199/63/004/002/004/013

The problem of constructing an optimal controller minimizing the integral mean square error is studied in a linear control system described by the equation

$$\frac{dx}{dt} = Ax(t) + \mu Bx(t-h) + m \xi, \quad (1)$$

where  $x$  is an  $n$ -dimensional deviation vector between the prescribed motion  $\{x_1 = 0\}$  and the true motion  $\{x_i(t)\}$  ( $i = 1, 2, \dots, n$ ),  $A$  and  $B$  are coefficient matrices,  $m$  is a vector,  $h > 0$  is the delay in action, and  $\mu$  is a small parameter characterizing the smallness of the delayed action. The establishment of the control  $\xi$  is carried out at any time instant  $t$  on the basis of information concerning the realization of the deviation  $x(t + \theta)$  at  $-h \leq \theta \leq 0$ . The optimality criterion is presented in the form of a theorem formulated on the basis of 1) the method of the Lyapunov function modified for equations with delayed argument and 2) concepts of the dynamic programming method. The problem of constructing an optimal

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OPTIMAL CONTROL [Cont'd]

8/199/63/004/002/004/013

controller is reduced to determination of the functionals  $v[x(\theta)]$  and  $\bar{\xi}^0[\dot{x}(\theta)]$  from equations obtained from the optimality criterion. Expressions for  $v$  and  $\bar{\xi}^0$  are derived in the form of infinite series in powers of  $\mu$ . It is proved that these series are convergent for sufficiently small values of  $\mu$ . [LK]

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L 11597-63

EWI(d)/FCC(w)/BDS AFFTC/APGC/ASD Pg-4/Pk-4/P1-4/  
Po-4/Pq-4 IJP(C)/BC

ACCESSION NR: AP3001140

S/0159/63/004/003/0622/0631

AUTHOR: Krasovskiy, N. N.

TITLE: On optimal control with random load

SOURCE: Sibirskiy matematicheskiy zhurnal, v. 4, no. 3, 1963, 622-631

TOPIC TAGS: random load, optimal control, linear system control

ABSTRACT: The control system described by the vector-form equation /6

$$\frac{dx}{dt} = f[t, x, w, u] + y(t) \quad (1)$$

is taken, where  $x(t)$  is an  $n$ -dimensional vector of generalized coordinates of the system subjected to a load described by Markov's random function  $w(t)$  and noise  $y(t)$ , and  $u(t, x, w)$  is the control vector. The problem at hand is that of constructing a control function  $u(t, x, w)$  which minimizes the performance functional

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ACCESSION NR: AP3001140

$$J[x_0, w_0; u] = M \left\{ \int_0^T \omega[t, x(t, u), u[t, x(t, u), w(t)]] dt + \sum_{s=1}^k \omega_s[x(t_s, u)] \right\} / x(0) = x_0, w(0) = w_0, \quad (2)$$

where  $\omega, \omega_s$  are given continuous functions,  $T > 0$ ,  $t_s$  are moments of time taken from the interval  $(0, T)$ , and  $M(\alpha/L)$  designates the conditional mathematical expectation of a random variable  $\alpha$ . The optimality criterion is defined and proved by a method based on the theory of Lyapunov functions and the principles of dynamic programming. It is proved that when two given functions  $\gamma(t, x, w)$  and  $u^0(t, x, w)$  are such that they satisfy a certain two conditions, then  $u^0(t, x, w)$  is an optimal control function and  $\gamma(0, x_0, w_0) = J[x_0, w_0, u^0]$ . The function  $\gamma(t, x, w)$  is called the optimal function of Lyapunov. In order to determine  $u^0$ , the function  $\gamma$  must be first determined as a solution of a partial differential equation, analogous to the Hamilton-Jacobi equation, derived from optimality conditions. There are only a few examples of an effective solution. The construction of the optimal control  $u^0$  minimizing a performance functional

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ACCESSION NR: AP3001140

of quadratic form is presented for the linear control system. Equations for  $Y$  and  $u^0$  are solved effectively, and an explicit form of an optimal control function is given. Orig. art. has: 28 formulas.

ASSOCIATION: none

SUBMITTED: 09Nov62

DATE ACQ: 01Jul63

ENCL: 00

SUB CODE: MM

NO REF SOV: 011

OTHER: 003

ch/ae  
Card 3/3

KRASOVSKIY, N.N. (Sverdlovsk); OSIPOV, Yu.S. (Sverdlovsk)

Stabilization of the motion of a controlled object with delay  
in a control system. Izv. AN SSSR. Tekh. kib. no.6:3-15 N-D  
'63. (MIRA 17:4)

KRASOVSKIY, N.N. (Sverdlovsk)

Optimum control with feedback signal delay. Avtom. i telem.  
24 no.8:1021-1036 Ag '63. (MIRA 16:8)

(Automatic control)

*Krasovskiy, N.N.*

AID Nr. 983-13 5 June

A PURSUIT PROBLEM (USSR)

Krasovskiy, N. N. Prikladnaya matematika i mekhanika, v. 27, no. 2,  
Mar-Apr 1963, 244-254. S/040/63/027/002/005/019

Two motions in n-dimensional space are studied: the pursued motion  $z(t)$  described by the equation

$$\frac{dz}{dt} = C(t)z + g(t) + d(t)v,$$

and the pursuing motion  $y(t)$  described by the equation

$$\frac{dy}{dt} = A(t)y + f(t) + b(t)u,$$

where  $y$  and  $z$  are vectors of phase coordinates  $y_1$  and  $z_1$  of the controlled objects,  $A(t)$  and  $C(t)$  are square matrices,  $b(t)$  and  $d(t)$  are parameters of

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AID Nr. 983-13 5 June

A PURSUIT PROBLEM [Cont'd]

S/040/63/027/002/005/019

the system in vector form,  $f(t)$  and  $g(t)$  are vectors of the external action, and  $u$  and  $v$  are scalars representing the control action. Under three types of constraint upon the control functions  $y(t)$  and  $z(t)$ , the problem of the optimal encounter of two motions is defined as follows: The optimum control functions  $u^0(t)$  and  $v^0(t)$ , with initial conditions to  $t_0$ ,  $y^0$ ,  $z^0$ , are to be determined such that the instant  $t_1$  of the first encounter of two motions is optimal. It is noted that  $t_1$  is difficult to determine, and the estimation of the value  $T \geq t_1 > t_0$ , called the instant of the first absorption of the process  $z(t)$  by the process  $y(t)$ , is studied. A theoretical equation is derived from which  $T$  could be determined in a series of cases, but the practical solution of this equation is complicated. One particular case of determining  $T$  and constructing the optimal control functions is analyzed. The problem is studied of constructing the control  $u(t)$  from the information obtained during the realization of  $y(t)$  and  $z(t)$  which brings the motions  $y(t)$  and  $z(t)$  together when the control  $v(t)$  of the object  $y(t)$  is unknown. Conditions are established under which such  $u(t)$  can be constructed.

[LK]

Card 2/2

L 18040-63

EWT(d)/BDS AFFTC/ASD/APGC Pg-4/Fk-4/Pl-4/Po-4/Pq-4

BC

ACCESSION NR: A3004111

S/0040/63/027/004/0641/0663

AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: Stabilization of unstable motions by additional forces with limited feedback

SOURCE: Prikladnaya matematika i mekhanika, v. 27, no. 4, 1963, 641-663

TOPIC TAGS: stability, control, differential equation, limited feedback

ABSTRACT: The author is interested in stabilization problems (in the sense of Lyapunov) in which the feedback is limited because measurements can be made only of certain functions of the state of the system. He gives various sufficient conditions (which are too complicated to state here) for existence of a solution to the problem. He exhibits differential equations whose solutions are the desired controls in the various cases above. These are all related to cases in which linearization of the original governing system leads legitimately to a solution of the problem. The author thanks V. E. Tret'yakov and Yu. Sh. Gurevich.

Orig. art. has: 109 formulas and 3 figures.

ASSOCIATION: none

SUBMITTED: 16Apr63

DATE ACQ: 15Aug63

ENCL: 00

SUB CODE: MM

NO REF SOV: 027

OTHER: 003

Card 1/1

GAL'PERIN, Ye.A; KRASOVSKIY, N.N.

Stabilization of steady motions of nonlinear controlled systems.  
Prikl. mat. i mekh. 27 no.6:988-1004, N-D '63. (MIRA 17:1)

17791-65 EMO-2/ENT(d)/FS3-2/EEC(k)-2/EMG(v)/EED-2/FS(b) Pe-5/Pg-4/Pk-4/  
 PL-4/Pn-4/Po-4/Pq-4 BC  
 ACCESSION NR: AP4048834 S/0280/64/000/005/0156/0164

AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: Gyroscopic stabilization of a controlled conservative mechanical system

SOURCE: AN SSSR. Izv. Tekhnicheskaya kibernetika, no. 5, 1964, 156-164

TOPIC TAGS: gyroscopic stabilization conservative mechanical system, Lagrange equation, gyroscopic stability

ABSTRACT: This article presents an analysis of the gyroscopic stabilization of a holonomic, conservative mechanical system with  $n$  degrees of freedom described by the Lagrange equations

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Pi}{\partial q_i} = b_i u \quad (i = 1, \dots, n), \quad (1)$$

where  $q_i$  are generalized coordinates,  $T$  and  $\Pi$  are the kinetic and

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1/3

L 17791-65

ACCESSION NR: AP4048834

potential energies, respectively,  $u$  is the control function (force), and  $b_1$  are functions defining the direction of the control  $u$ . The problem consists in selecting a control function  $u(q, \dot{q})$  for which the state of equilibrium is asymptotically stable. The concept of stabilization by a first approximation is defined and the stabilization problem is reduced to the controllability of the first approximation system. It is pointed out that when a first approximation system is not controlled completely by the control  $u$ , that is, the system described by (1) can not be stabilized by a first approximation, then it is possible to achieve complete controllability of the first approximation by applying additional gyroscopic forces to (1). It is shown that gyroscopic stabilization has features in common with known properties of the gyroscopic stability investigated by N. G. Chetayev. Conditions are established when an unstable conservative system can be stabilized by means of control forces of a general nature by applying the gyroscopic effect and can not be stabilized by dissipative forces. It is pointed out that the gyroscopic effect can be utilized in controlling mechanical systems with a large number of degrees of freedom when the original design of the controlled system does not assure the complete controllability of the system. Orig. art. has: 22 formulas.

Card

2/3

L 17791-65

ACCESSION NR: AP4048814

ASSOCIATION: none

SUBMITTED: 04May64

ENCL: 00

SUB CODE: NG, MA

NO REF SOV: 022

OTHER: 000

ATD PRESS: 3153

Card 3/3

ACCESSION NR: AP4042490

S/0103/64/025/007/1047/1057

AUTHOR: Al'brekht, E. G. (Sverdlovsk); Krasovskiy, N. N.  
(Sverdlovsk)

TITLE: Observation of a nonlinear control system in the neighborhood  
of a given motion

SOURCE: Avtomatika i telemekhanika, v. 25, no. 7, 1964, 1047-1057

TOPIC TAGS: nonlinear control system, quasilinear control system,  
control system phase coordinate, phase coordinate determination,  
first approximation system, control system position determination

ABSTRACT: To construct or stabilize the optimum control system, it  
is necessary to have complete information concerning the position  
of the control system at any instant in a phase space. However,  
direct measurement of phase space coordinates is difficult or  
impossible in many cases. The authors present a method for deter-  
mining the position of the nonlinear control system when there are  
small deviations of the motion of the system. A similar procedure  
is also applied to determine the position of a quasi-linear control

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ACCESSION NR: AP4042490

system provided the prescribed motion and all possible deviations from it are located in a certain bounded region. Assuming that observation of the motion described by the first approximation system is possible, the solution of the problem is derived in the form of a series in powers of a small parameter and of certain functionals. Conditions are established for the convergence of the series. The solution of linear differential and algebraic equations is needed to complete the determination of the position of the control system. Orig. art. has: 34 formulas.

ASSOCIATION: none

SUBMITTED: 20Jul63

ATD PRESS: 3071

ENCL: 00

SUB CODE: MA

NO REF SOV: 013

OTHER: 002

Card 2/2

ACCESSION NR: AP4013376

S/0040/64/028/001/0003/0014

AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: Theory of controllability and observability of linear dynamic systems

SOURCE: Prikladnaya matematika i mekhanika, v. 28, no. 1, 1964, 3-14

TOPIC TAGS: controllability, observability, linear system, dynamic system, correction signal, optimal process, Pontryagin maximum principle, dynamic programming, programming

ABSTRACT: The author is interested in two problems in the theory of optimal processes: that of control (the choice of forces which transfer a controlled object from one state to another) and that of observation (the operations which determine the unknown moving coordinates of an object consistent with the observed value of the variable). One approach to these linear problems is related to the theory of the L-problem of moments (see N. Akhiezer and M. Kreyn, O nekotorykh voprosakh teorii momentov, CONTI-NTVU, 1938, 171). The idea of this approach is that problems of computing controlling forces or correction signals are interpreted as problems of constructing linear functionals which assume given values associated with certain known elements. Thus the problem must

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ACCESSION NR: AP4013376

be transformed in such a way that either the restrictions or minimized estimates are expressed in terms of the norm of a desired functional. On the basis of the observation problem, the author interprets from the point of view of a duality principle those relations which determine optimal solutions of the two problems above, approaching them as L-problems of moments. Orig. art. has: 53 formulas.

ASSOCIATION: none

SUBMITTED: 24 Oct 63

ATD PRESS: 3059

ENCL: 00

SUB CODE: MA

NO REF SOV: 009

OTHER: 002

Card 2/2

GABRIYELIAN, M.S.; KRASOVSKIY, N.N.

Problem of stabilization of a mechanical system. Prikl. mat. i  
mekh. 28 no.5:801-811 S-O '64.

(MIRA 17:11)

KARLOVSKY, H.H. (Karlavskiy)

Approximation of a problem involving the construction of regulators in a system of related arguments. (1971. ent 1 mekh. 20 no. 1971-11 (1971) 1971-1971)

L 17667-65

BXP/EWT(1) IJP(c)

ACCESSION NR: AP4046264

S/0040/64/028/005/0801/0811

AUTHOR: Gabriyelyan, M. S.; Krasovskiy, N. N.

TITLE: On the problem of stabilization of a mechanical system <sup>B</sup>

SOURCE: Prikladnaya matematika i mekhanika, v. 28, no. 5, 1964, 801-811

TOPIC TAGS: mechanical system stabilization, holonomic system, system control, regulator analytic construction dissipative force, gyroscope force, asymptotic stability

ABSTRACT: A study was made of the stabilization of a holonomic mechanical system by additional forces in the neighborhood of the state of equilibrium. The motion of the system is described by equations

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i(t, q_1, \dots, q_n, u) \quad (i=1, \dots, n) \quad (1)$$

where  $q_i$  are curvilinear coordinates,  $T$  is the kinetic energy,  $Q_i$  are generalized forces, and  $u$  is the control action under consideration.

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L 17667-65

ACCESSION NR: AP4046264

action. Under the assumptions that (1) has a solution  $q_i = 0$  at  $u \equiv 0$  and that its linear approximation is stationary and is of the form

$$\sum_{j=1}^n a_{ij} q_j'' = \sum_{j=1}^n b_{ij} q_j + b_{i0} u \quad (i=1, \dots, n) \quad (2)$$

where  $a_{ij}$ ,  $b_{ij}$ ,  $b_{i0}$  are constants, the form  $\sum_{i,j} a_{ij} q_i q_j$  is positive definite and  $b_{ij} = b_{ji}$  (such a linear system is called conservative), the following problem of analytic construction of optimal regulators is formulated; to find a control function  $u$  depending only on the velocities  $q_1'$ ,  $q_2'$ , ...,  $q_n'$ , which minimizes a certain performance functional and assures the asymptotic stability of the motion  $q_i = 0$ . Various cases are analyzed and conditions are established under which the solution of this problem is possible. The problem of the stabilization of motion by dissipative forces is analyzed and its connection with the problem of the analytic construction of optimal regulators is established. It is shown that conditions for stabilization by dissipative forces coincide with general conditions for control and stabilization of the system. The effects of dissipative

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ACCESSION NR: AP4046264

and gyroscopic forces upon the controllability, observability, and stability of conservative mechanical systems in the neighborhood of the state of equilibrium are studied. It is shown that by proper selection of dissipative and gyroscopic forces the quality of controllability, observability, and stability can be improved in many cases. Orig. art. has: 49 formulas.

ASSOCIATION: none

SUBMITTED: 18May64

ENCL: 00

SUB CODE: MA

NO REF SOV: 016

OTHER: 002

Card 3/3

L 7695-66 EWT(d)/EWP(v)/EWP(k)/EWP(h)/EWP(l)

ACC NR: AP5028761

SOURCE CODE: UR/0376/65/001/011/1415/1427

AUTHOR: Krasovskiy, N. N.

ORG: Sverdlovsk Branch of the Mathematics Institute im. V. A. Steklova  
(Sverdlovskoye otdeleniye matematicheskogo instituta)

65  
B

TITLE: On optimum control at discrete feedback signals

SOURCE: Differentsial'nyye uravneniya, v. 1, no. 11, 1965, 1415-1427

TOPIC TAGS: optimum control theory<sup>14</sup>, signal optimum frequency, control function, dynamic programming

ABSTRACT: A control system described by the equation

$$dx = f(x, u)dt + dv, \quad (1)$$

where  $x$  is a vector of the phase coordinates of the control system,  $u$  is a control vector,  $f$  is a given vector function characterizing the dynamic properties of the control system and  $dv$  is a variable-intensity disturbance of a white noise type, is studied under the assumption that information concerning the current state  $x(t)$  of the control system enters the controlling device at certain discrete instants of time  $\tau_s$ . Two cases are investigated: 1) the sequence  $\{\tau_s\}$  is determined and the instants  $\tau_s$  follow each other at equal intervals of time  $\Delta\tau = 1/\lambda$ ; 2) the sequence  $\{\tau_s\}$  is ran-

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L 7695-66

ACC NR: AP5028761

dom and instants  $\tau_s$  are distributed according to the Poisson law with the frequency  $\lambda$ . The optimum problem investigated in the article is formulated as follows: from the sets of  $\lambda > 0$  and allowable control functions  $u$ , we are to select the optimal frequency  $\lambda^0$  and the optimal control  $u^0$  which minimize the functional

$$I = \lim_{T \rightarrow \infty} \frac{M(\xi_T)}{T} \quad (2)$$

where  $M(\xi_T)$  is the mathematical expectation of the random value  $\xi_T$  which characterizes the performance of the system (1) on the interval  $[0, T]$  and is given by the expression

$$\xi_T = \int_0^T \omega[x(t), u(t)] dt, \quad (3)$$

where  $\omega[x, u]$  is a given function defined by the chosen performance criterion. Bellman's method of dynamic programming is applied to this problem. General expressions for determining  $u^0$  and  $\lambda^0$  are derived in terms of the functional  $U_{T, \lambda}$  of the method of dynamic programming. It is stressed that no construction of the functional  $U_{T, \lambda}$  in closed form is known for the general case. Therefore, the author seeks the solution of the problem for a particular case, when system (1) is linear and the function  $\omega$  in formula (3) is of quadratic form. Under the assumption that the linear system satisfies the conditions of asymptotic stability, the condition necessary for

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ACC NR: AP5028761

solving the optimal problem is established and a method for determining the optimal frequency  $\lambda^0$  and the optimal control  $u^0$  in closed form is presented. An example illustrates the theory. Orig. art. has: 68 formulas. [LK]

SUB CODE: MA, ME/ SUBM DATE: 15Jun65/ ORIG REF: 009/ OTH REF: 001/ ATD PRESS:

4142

Card 3/3

L 58546-65 ENT(d)/EPF(n)-2/EWP(1) Po-4/Pq-4/Pg-4/Pae-2/Pu-4/Pk-4/Pl-4 IJP(c)  
ACCESSION NR: AP5012880 UR/0280/65/000/002/0102/0109 WH/BC

AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: Stabilizing systems in which noise depends on the actuating signal

SOURCE: AN SSSR. Izvestiya. Tekhnicheskaya kibernetika, no. 2, 1965, 102-109

TOPIC TAGS: automatic control, automatic control design, automatic control system, automatic control theory

ABSTRACT: The problem is considered of the actuating signal which stabilises an uncertain movement of the plant when a random noise arises in the course of control, the noise intensity increasing with the actuating force. The control law  $u = u[x]$  is found and analyzed which makes the motion  $x \approx 0$  of the system

describable by  $\frac{dx}{dt} = Ax + bu + c \frac{dw}{dt}$  probabilistically asymptotically stable. In this stochastic vector differential equation:  $x$  is the  $n$ -variable vector of phase

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ACCESSION NR: AP5012880

coordinates  $x_1$  of the plant;  $u$  is the scalar actuating signal;  $b$ ,  $c$  are the constant vectors;  $A$  is the constant  $(n \times n)$ -matrix;  $d\varphi/dt$  is the random noise. The solution of this problem is based on the theory of analytical designing of optimal controllers. A method for numerical evaluation of the noise intensity is indicated. Orig. art. has: 37 formulas.

ASSOCIATION: none

SUBMITTED: 20Jun64

ENCL: 00

SUB CODE: DP, IE

NO REF SOV: 007

OTHER: 002

Card *llc*  
2/2

L 52365-65 EWT(d) Pg-4 IJP(c)

ACCESSION NR: AP5012439

UR/0376/65/001/002/0143/0152

AUTHOR: Krasovskiy, N. N.

15  
14  
B

TITLE: A property of a linear stable system completely controlled by a random influence

SOURCE: *Differentsial'nyye uravneniya*, v. 1, no. 2, 1965, 143-152

TOPIC TAGS: random process, differential equation /C

ABSTRACT: The author considers the vector linear asymptotically stable differential equation

$$\frac{dx}{dt} = Ax + b\eta, \quad (1)$$

where  $x$  is an  $n$  vector of phase coordinates,  $A$  is an  $n \times n$  constant matrix,  $b$  is a constant  $n$  vector, and  $\eta$  is a scalar random function of time,  $\eta(t)$  being assumed to be a sequence of random impulses

$$\eta(t) = \sum_{j=1}^{\infty} \gamma_j \delta(t - \tau_j). \quad (2)$$

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ACCESSION NR: AP5012439

occurring with frequency  $\lambda$  governed by a Poisson law. In the case where

$$\frac{dx}{dt} = Ax + bu, \quad (3)$$

is completely controlled, he shows that with probability arbitrarily close to 1, with the growth of time motion of the system will become arbitrarily close to any point  $x^*$  of the phase space. If (3) is incompletely controlled, this property is possessed only by points  $x^*$  from some linear subspace  $W$  of the phase space. Orig. art. has: 56 formulas.

ASSOCIATION: Sverdlovskoye otdeleniye Instituta matematiki im. V. A. Steklova  
(Sverdlovsk Division of the Institute of Mathematics)

SUBMITTED: 21Nov64

ENCL: 00

SUB CODE: MA

NO REF SOV: 006

OTHER: 001

2/2

KRASOVSKIY, N.N.

Optimum control at discrete feedback signals. Dif. urav. 1  
no.11:1415-1427 N '65.

(MIRA 18:12)

1. Sverdlovskoye otdeleniye matematicheskogo instituta imeni  
V.A.Steklova. Submitted June 15, 1965.

KRASOVSKIY, N.N.

Problem concerning the observation of a linear dynamic system,  
and equations with delayed argument. Dif. urav. 1 no. 12:  
1551-1556 D '65. (MIRA 18:12)

1. Matematicheskiy institut imeni Steklova, Sverdlovskoye  
otdeleniye. Submitted June 15, 1965.

L 65247-65 EWT(d)/EPF(n)-2/T/EWP(1) IJP(c) WW/BC  
 UR/0280/65/000/004/0003/0013  
 ACCESSION NR: AP5021846  
 AUTHOR: Krasovskiy, N. N. (Sverdlovsk); Repin, Yu. N. (Sverdlovsk); Tret'yakov, V. Ye. (Sverdlovsk)  
 TITLE: Certain game situations in control theory

SOURCE: AN SSSR. Izvestiya. Tekhnicheskaya kibernetika, no. 4, 1961, 3-13

TOPIC TAGS: automatic control theory, automatic control system, game theory

ABSTRACT: The problem of pursuit of one controlled object by another represents an important branch of the modern theory of control systems. The law of control is usually chosen in such a way that the corresponding algorithm materializing the control commands can be conveniently realized. However, this type of approach utilizing an a priori chosen control law does not always result in an optimum pursuit utilizing inexpensive means. To invert the problem and start from the optimization requirement the authors view the pursuit problem as a game between two players one of which tries to reduce and the other to increase a certain index characterizing the result of the game. Here the first player represents the control system while the second player represents the pursued object. The control laws represent the strategy of the game and the time of encounter is the characterization index. This article describes a particular method for the choice of controls, and the general results are illustrated on examples of simple mechanical

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ACCESSION NR: AP5021846

motion modeled on computers (motion of two mass points along a horizontal axis subject to control forces  $u$  and  $v$  limited by the conditions

$$\int_0^T u^2(t) dt \leq \mu^2(\tau), \quad \int_0^T v^2(t) dt \leq \nu^2(\tau).$$

at the instant of encounter one requires the equality of velocities as well as of positions). Orig. art. has: 36 formulas and 10 figures.

ASSOCIATION: None

SUBMITTED: 00

ENCL: 00

SUB CODE: IE, MA

NO REF SOV: 002

OTHER: 002

Card 2/2

L 63231-65 EWT(d)/EPF(n)-2/EWP(v)/EWP(k)/EWP(h)/EWP(l) Po-4/Pq-4/Pf-4/Pg-4/Pu-4/  
 Pk-4/Pl-4 IJP(c) KW/EC UR/0242/65/020/003/0153/0174  
 ACCESSION NR: AP5015384 519.9

AUTHOR: Krasovskiy, N. N.

TITLE: Optimal control in ordinary dynamic systems

SOURCE: Uspekhi matematicheskikh nauk, v. 20, no. 3, 1965, 153-174

TOPIC TAGS: differential equation system /6

ABSTRACT: N. N. Krasovskiy, a prominent Soviet specialist in the theory of automatic control and stability, presents an interesting survey and evaluation of problems in the theory of dynamic systems described by the system of differential equations written in vector form

$$\dot{x} = f(t, x, u). \quad (1)$$

Two aspects of the problem of optimal control are considered: a) Programmed control, when information concerning the initial state  $x_0(t)$  ( $t = 0$ ) is given and it is necessary to find the control function  $u(t)$  ( $0 \leq t \leq T$ ) which takes the sys-

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L 62261-65

ACCESSION NR: AP5015384

item at the end of the process ( $t = T$ ) into a given final state  $x_1(t)$  and optimizes the given performance functional  $I[x(t), u(t)]$ . b) The problem of the synthesis of an optimal system with feedback. Here the law of optimal control is sought in the form of equations connecting the control signal  $u(t)$  with the values of  $y$  given by equations of the form  $y = \varphi(x, u)$  or  $y = \varphi(x, u)$  which carry information concerning the current state of the control system. Several particular examples of problems a) and b) are presented. It is indicated that the most complete studies and final results pertain to the development of necessary optimality criteria for programmed control of systems (1). A short history of the development of this theory in the 1940's and 1950's is presented. It is stressed that in this period, research workers strongly felt the necessity of having a broad, well-founded optimality criterion convenient for practical applications. Such a criterion was formulated and its theory developed in 1956—1960 by L. S. Pontryagin and his co-workers and is well known as the "maximum principle." N. N. Krasovskiy asserts that the maximum principle has satisfied the urgent needs of optimal control theory because it has provided a general rule for setting up the variational equations. He considers that the theory developed by L. S. Pontryagin and his co-workers is a very impor-

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ACCESSION NR: AP5015384

tant step in the theory of optimal processes. The other important contribution of L. S. Pontryagin and his co-workers rests in the fact that their profound results have attracted the attention of many scientists to optimal control problems. It is noted that the work done by Pontryagin and his co-workers was stimulated by the optimal problem formulated by A. A. Fel'dbaum. The author surveys a number of studies on various interpretations and generalizations of the maximum principle and also on deriving of analogous criteria by classical methods of variational calculus.

The author analyzes and evaluates the second approach to the solution of optimal problems aimed mostly at problems of synthesizing optimal systems with feedback. This is the method of dynamic programming developed by R. Bellman and his co-workers. This method reduces the solution of optimal problems to the solution of Hamilton-Jacobi partial differential equations of the form

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ACCESSION NR: AP5015384

$$\min_u H \left[ \frac{\partial v}{\partial x}, x, u \right] = 0 \quad (2)$$

where

$$v(x_0) = \min \int_0^T f^0 dt,$$

$$H = \sum_{i=1}^n \frac{\partial v}{\partial x_i} f^i + f^0.$$

The author stresses that critics cite the following shortcomings of this method: equation (2) is not a mathematical theorem, because, in contrast to the maximum principle of Pontryagin, where the existence of the continuous non-zero vector function  $\psi(x)$  is asserted, the existence of the smooth function

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L 63231-63

ACCESSION NR: AP5015384

$v(x)$  is not proved here. However, it is known that for some simple typical problems,  $v(x)$  is not a smooth function and in this case it is difficult to retain the optimality criterion in form (2). In general, the critics claim carelessness in the work of Bellman and his co-workers. Bellman's optimality principle, asserting that a segment of an extremal is also an extremal and the corresponding study of the elementary process of the propagation of optimal motion are well known facts. Although N. N. Krasovskiy agrees that it is hard to question the correctness of these remarks, he considers Bellman's highest contribution to be the fact that he was the first systematically to apply the apparatus of equations (2) to optimal control problems, he developed it for a large number of problems in the synthesis of optimal control systems, he presented useful recommendations for setting up the functional equations in form (2), and he developed computational schemes for their solution (mostly for a discrete time  $t_j$  ( $j = 1, 2, \dots, N$ )). Although the methods of dynamic programming have not yet taken the form of a complete mathematical theory, they

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L 63231-66

ACCESSION NR: AP5015384

constitute a very good guide in the synthesis of optimal control systems. The real inconvenience of this method is the solution (desirable in closed form) of difficult equations (2) and of other functional relations. The author surveys the particular classes of optimal control problems where the method of dynamic programming has been successfully applied.

N. N. Krasovskiy asserts that development of the fundamental mathematical theory of necessary optimality criteria and the success of this theory initially overshadowed other important problems of optimal control. Such problems surveyed in this article are: a) the existence of allowable controls which take the system from the initial state to the final state; b) the existence of optimal control in the class of allowable controls; c) the controllability and observability of linear, quasi-linear, and nonlinear dynamic systems; d) stabilization of control systems. Orig. art. has: 37 formulas.

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L 63231-65

ACCESSION NR: AP5015384

ASSOCIATION: none

SUBMITTED: 00

ENCL: 00

SUB CODE: IE, MA

NR REF SOV: 0044

OTHER: 015

ATD PRESS: 4046-F

Card 7/7

L 48338-65

EMT(d) Pg-4 IJP(c)

ACCESSION NR: AP5010627

UR/0040/65/029/002/0218/0225

AUTHOR: Krasovskiy, N. N. (Sverdlovsk)

TITLE: Problem of damping a linear system with minimal control intensity

SOURCE: Prikladnaya matematika i mekhanika, v. 29, no. 2, 1965, 218-225

TOPIC TAGS: differential equation, optimal control, energy dissipation

ABSTRACT: Let  $x$  be an  $n$ -vector of phase coordinates  $\{x_i\}$  of an object,  $u \in U$  an  $r$ -vector of controlling forces  $\{u_j\}$ ,  $A$ ,  $B$  be  $n \times n$  and  $n \times r$  matrices  $\{a_{ij}\}$  and  $\{b_{ij}\}$ . Let the initial state  $x^0$  of

$$\dot{x}/dt = Ax + Bu \quad (1)$$

be given. The problem is to find a control  $u^0(t)$  which takes (1) from  $x(0) = x^0$  to  $x(T) = 0$  and for which

$$J[u^0(\tau)] = \min_u J[u(\tau)] \text{ for } u \text{ in } U \quad (2)$$

where  $J[u(\tau)]$  ( $0 < \tau < T$ ) is an estimate of the quality of the control. The form of the criterion can sometimes simplify solution, since it often does not require the

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ACCESSION NR: AF5010627

construction of a space  $\{h(\tau)\}$  with norm  $\rho[h]$  such that  $\rho*[u] = \xi[u]$ . As  
an illustrative example, the author treats the problem of damping of a linear  
oscillator  $x'' + K^2 x = u$ . Orig. art. has: 52 formulas.

ASSOCIATION: none

SUB CODE: MA

SUBMITTED: 19Nov64

ENCL: 00

NO REF SOV: 003

OTHER: 001

Card 2/2

L 57504-85 ENT(d)/EMP(v)/EMP(k)/EMP(h)/EMP(l) Pf-I IJP(c)

ACCESSION NR: AP5014931

UR/0040/65/029/003/0401/0407

AUTHORS: Krasovskiy, N. N. (Sverdlovsk); Sheloment'yev, G. S. (Sverdlovsk)

TITLE: On correcting the motion of a system with two degrees of freedom at one cyclic coordinate

SOURCE: Prikladnaya matematika i mekhanika, v. 29, no. 3, 1965, 401-407

TOPIC TAGS: Lagrange equation, control theory, control research, control system

ABSTRACT: The problem of constructing a control force  $u(t)$  guiding a system with two degrees of freedom is analyzed. The control system is described by the Lagrange equations

$$f(q_1', q_1'', q_1, p_1) = b_1(q_1, q_1', p_1) u, \quad dp_1/dt = b_2(q_1, q_1', p_1) u,$$

where  $q_1$  are generalized coordinates,  $T(q, q')$  is kinetic energy,  $\Pi(q)$  - potential energy,  $b_1(q, q')$  are functions defining the direction of the external control force, and  $u(t)$  is the value of the control force. The problem consists of selecting  $u(t)$  which leads the system

$$\frac{d}{dt} \left( \frac{\partial T}{\partial q_1'} \right) - \frac{\partial T}{\partial q_1} = - \frac{\partial \Pi}{\partial q_1} + b_1 u \quad (i=1, 2)$$

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ACCESSION NR: AP5014931

toward the given motion  $q_1 = q_1^0 = \text{const}, \quad \dot{q}_1 = 0, \quad p_1 = p_1^0 = \text{const}.$

The first approach discussed involves a linear approximation. The given equation set is converted to a first order approximation form. Necessary and sufficient conditions for controlling the system are stated. The method of solution of the problem follows the procedures outlined by N. N. Krasovskiy (Ob odnoy zadache optimal'nogo regulirovaniya. PMM 1957, t. 21, vyp. 5). The solution leads to the derivation of a general form of the control force which is a function of parameters found in the manner described by the author. Further consideration is given to the nonlinear system case. An iterative process is described for constructing the control function which solves the nonlinear problem. A concrete example of the solution method is given for the case of a mathematical pendulum constrained as indicated in Fig. 1 on the Enclosure. Orig. art. has: 4 figures and 37 equations.

ASSOCIATION: none

SUBMITTED: 03Feb65

ENCL: 01

SUB CODE: ME, MA

NO REF SOV: 004

OTHER: 001

Card 2/3

KRASOVSKIY, N.N. (Sverdlovsk); REPIN, Yu.M., (Sverdlovsk); TRET'YAKOV, V.Ye.  
(Sverdlovsk)

Some game situations in the theory of regulated systems.

Izv. AN SSSR. Tekh. kib. no.4:3-13 J1-Ag '65.

(MIRA 18:11)

L 16058-66 EWT(d) IJP(c)

ACC NR: AP6004067

SOURCE CODE: UR/0040/65/029/005/0828/0834

AUTHORS: Bondarenko, V. I. (Nizhniy Tagil); Krasovskiy, N. N. (Sverdlovsk);  
Filimonov, Yu. M. (Sverdlovsk)

ORG: none

TITLE: The problem of putting to rest a linear system

SOURCE: Prikladnaya matematika i mekhanika, v. 29, no. 5, 1965, 828-834

TOPIC TAGS: differential equation, optimal control, steepest descent

ABSTRACT: The authors consider a controlled system described by the linear vector  
differential equation

$$\frac{dx}{dt} = Ax + Bu \quad (1)$$

where  $x$  is an  $n$ -dimensional vector of the phase coordinates of the controlled object and  $u$  describes the controlling influence. By the method of steepest descent they solve the problem of choosing the (optimal) control  $u^0(t)$  which in given time  $T$  takes (1) from state  $x_0$  to state  $x(T)$  minimizing

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ACC NR: AP6004067

$$J(u) = \max \left\{ \max_{\tau} |u(\tau)|, \theta \int_0^T |u(\tau)| d\tau \right\} = \min_{\theta = \text{const}} \quad (2)$$

They treat several specific examples. Orig. art. has: 4 figures and 30 formulas.

SUB CODE: 12/

SUBM DATE: 10Jun65/

ORIG REF: 003

Card 2/2

L 10691-66 EWT(d)/EWA(m)-2 IJP(c)

ACC NR: AP6000545

SOURCE CODE: UR/0040/65/029/006/1081/1083

AUTHOR: <sup>55/54</sup> Bulgakov, N. G. (Minsk); Krasovskiy, N. N. (Sverdlovsk)

ORG: none

TITLE: On stabilizing nonstationary systems

SOURCE: Prikladnaya matematika i mekhanika, v. 29, no. 6, 1965, 1081-1083

TOPIC TAGS: stability theory, nonstationary system stability, asymptotic stability, Lyapunov stability, control system stability

ABSTRACT: The article is a study of one particular case of stabilizing by first approximation a nonstationary control system. The control system whose phase coordinate  $y$  is subjected to small disturbances  $x$  is taken and the disturbed motion is described by the following system of equations:

$$\dot{x} / dt = A(t) x + B(t) u + g(t, x, u) \quad (1)$$

where  $A(t)$  is an  $n \times n$  matrix,  $B(t)$  is an  $n \times m$  matrix,  $u$  is an  $m$ -dimensional control vector, and  $g$  is a vector function whose terms are of order higher than one with respect to  $x$  and  $u$  and are uniform with respect to  $t$  when  $0 \leq t \leq \infty$ . The stabilization problem is defined as follows: to find a control function  $u(x, t)$  which ensures Lyapunov's asymptotic stability of the zero solution  $x = 0$  of equation (1). Under

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ACC NR: AP6000545

special condition that the matrix

$$V = (B(\tau), A(\tau)B(\tau), \dots, A^{n-1}(\tau)B(\tau)) \quad (2)$$

for every fixed value  $t - \tau = \text{const} > 0$  is of rank  $n$ . Its first linear approximation is considered along with the complete system (1). On the basis of results of the theory of controlling linear control systems, sufficient conditions are derived under which the nondisturbed motion of system (1) is stabilized by the linear control function  $u(x, t) = p(t)x$ , where  $P(t)$  is a certain matrix. Orig. art. has: 20 formulas. [LK]

SUB CODE: 12 A. SUBM DATE: 15Jun65/ ORIG REF: 008/ ATD PRESS: 4167

HW

Card 2/2

L 24770-66 EWT(d)/EWP(1) LIP(c)  
ACC NR: AP6015534

SOURCE CODE: UR/0376/65/001/001/0005/0016

AUTHOR: Krasovskiy, N. N.

ORG: Ural'sk State University im. A. M. Gor'ki (Ural'skiy gosudarstvennyy universitet)

TITLE: Stabilization of dynamic systems by supplemental forces The paper was presented at the first Belorussian Mathematical Conference held from 25 to 28 January 1964

SOURCE: Differentsial'nyye uravneniya, v. 1, no. 1, 1965, 5-16

TOPIC TAGS: dynamic system, motion stability, asymptotic property

ABSTRACT: This article is devoted to a discussion of some general results in the mathematical theory of stabilization of dynamic systems. Primary attention is given to two cases: 1) stabilization problems looked upon as a natural development of the overall problem of the stability of motion, 2) stabilization problems analysed in connection with control and observation problems in dynamic systems. The works of N. G. Chetayev and his school are used in applying the theory of motion stability to the problem analysed. A brief discussion of the results of the theory of control and observation in linear dynamic systems is presented.

A stabilization problem which is analyzed is formulated in its simplest form as the determination of a function  $u=Gx$  such that the movement  $x=0$  of the system whose movement is described by  $dx/dt=Ax+Bu$  will be asymptotically

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ACC NR: AP6015534

stable. The problem of stabilization of a nonlinear system is also analysed  
Finally, the results of the first types of systems are expanded to cover  
systems with infinite degrees of freedom. Orig. art. has: 5 formulas. [JPRS]

SUB CODE: 12 / SUM DATE: none / ORIG REF: 024 / OTH REF: 001

Card 2/2 *mf-5*

L 29828-66 EWT(d)/EWP(v)/EWP(k)/EWP(b)/EWP(l) BC

ACC NR: AP6011650

SOURCE CODE: UR/0020/66/167/003/0540/0542

AUTHOR: Krasovskiy, N. N. (Corresponding member AN SSSR)

ORG: Sverdlovsk Department of the Institute of Mathematics, im. V. A. Steklov,  
Academy of Sciences SSSR (Sverdlovskoye otdeleniye Matematicheskogo Instituta Akademii  
nauk SSSR)

TITLE: Approximation of one problem on optimal control in a system with residual  
effect

SOURCE: AN SSSR. Doklady, v. 167, no. 3, 1966, 540-542

TOPIC TAGS: control, control system, integral equation, matrix function, system  
analysis, operations research

ABSTRACT: A study of a problem in optimal control is made. The optimal control  
problem  $u^0[t, x(t + \theta)]$ , which stabilizes to an asymptotic level the system with the  
residual effect

$$\dot{x}(t) = \int_0^\theta d_0 H(t, \theta) x(t + \theta) + B(t) u,$$

and minimizes the integral

$$I_\infty[t_0, x_0(\theta), u] = \int_0^\infty [\| \dot{x}(t) \|^2 + \| u(t) \|^2] dt,$$

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L 29828-66

ACC NR: AP6011650

is studied. In these expressions  $x = \{x_i\}$  is the n-dimensional vector of phase coordinates of the controlled object;  $x(t + \theta)$  ( $-\sigma \leq \theta \leq 0$ ) is an element of the system trajectory, and  $\sigma > 0$  is a constant;  $H(t, \theta)$  is a matrix with bounded variation about  $\theta$  for  $-\sigma \leq \theta \leq 0$ ;  $B(t)$  is a matrix bounded for  $t \geq 0$ ;  $u$  is an r-dimensional vector of the control force;  $x_0(0)$  is an initial disturbance  $x(t_0 + \theta)$  ( $-\sigma \leq \theta \leq 0$ ) of the object. The optimal control  $u^0[t, x(t + \theta)]$  for this problem has the form of the linear functional

$$u^0[t, x(t + \theta)] = P(t)x(t) + \int_{-\sigma}^0 Q(t, \theta)x(t + \theta)d\theta.$$

This problem is solved by an approximation method approach. Portions of this approach are taken from earlier work by M. Ye. Salukvadze (Avtomatika i telemekhanika, 23, No. 2, 1962) and by N. N. Krasovskiy (PMM, 28, v. 4, 1964). The problem is restated with the definition of optimality criteria. Optimality conditions are established, and proofs of limiting relationships are developed. The form of the optimal control vector is shown. Orig. art. has: 9 equations.

SUB CODE: 12/ SUBM DATE: 18Oct65/ ORIG REF: 005

Card 2/2 *fv*

L 29186-66 EWT(d)/FSS-2 IJP(e) BC

ACC NR: AP6017846

SOURCE CODE: UR/0376/66/002/005/0587/0599

AUTHOR: Krasovskiy, N. N.; Tret'yakov, V. Ye.

ORG: Ural State University im. A. M. Gor'kiy (Ural'skiy gosudarstvennyy universitet)

TITLE: On the <sup>16</sup>pursuit problem<sup>9</sup> with constraints imposed upon the impulses of the control response

SOURCE: Differentsial'nyye uravneniya, v. 2, no. 5, 1966, 587-599

TOPIC TAGS: automatic control, optimal control, pursuit problem, motion mechanics

ABSTRACT: The pursuit problem for two objects whose motion is described by the differential equations:

$$\dot{y} = Ay + Bu, \quad (1)$$

$$\dot{z} = Gz + Mv. \quad (2)$$

where  $y(t)$ ,  $z(t)$  and  $u(t)$ ,  $v(t)$  are the state and control vectors of the pursuing and the pursued objects, respectively, and  $A, B, G, M$  are constant matrices, is analyzed under the assumption that  $u(t)$  and  $v(t)$  are constrained at every instant  $t$  by the

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